

# Relativistic Positioning Systems: current status<sup>1</sup>

Bartolomé Coll, Joan J. Ferrando and Juan A. Morales-Lladosa

*Departament d'Astronomia i Astrofísica,  
Universitat de València, 46100 Burjassot, València, Spain.  
bartolome.coll@uv.es, joan.ferrando@uv.es, antonio.morales@uv.es*

**Abstract.** A *relativistic positioning system* consists in a set of four clocks broadcasting their respective proper time by means of light signals. Among them, the more important ones are the *auto-located positioning systems*, in which every clock broadcasts not only its proper time but also the proper times that it receives from the other three. At this level, no reference to any exterior system (the Earth surface, for example) and no synchronization are needed. The current status of the theory of relativistic positioning systems is sketched.

**Keywords:** Coordinate systems, reference systems, positioning systems

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## INTRODUCTION

In astronomy, space physics and Earth sciences, the increase in the precision of space and time localization of events is associated with the increase of a better knowledge of the physics involved. Up to now all time scales and reference systems, although incorporating so-called ‘relativistic effects’<sup>2</sup> in their development, start from Newtonian conceptions.

Nowadays, a fully relativistic approach to Global Navigation Systems is becoming more and more urgent. A pioneering proposal to develop such a relativistic theory was presented at the ERE-2000 (XXIII Spanish Relativity Meeting celebrated in Valladolid) [1], and also in [2, 3, 4]. At the year 2005, the International School on *Relativistic Coordinates, Reference and Positioning Systems* took place at Salamanca, where the status of the theory at that time and work in progress were communicated and discussed [5, 6]. Since then, progress on the subject has been attained and published elsewhere [7]-[21].

In Relativity, the space-time is modeled by a four-dimensional manifold. In this manifold, most of the coordinates are usually chosen in order to simplify mathematical operations, but some of them, in fact a little set, admit more or less simple physical interpretations. What this means is that some of the ingredients of such coordinate

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<sup>1</sup> This paper is about the future interest and present state of some developments of the theory of relativistic positioning systems. For this reason, its structure has much of a potpourri of already published papers, all of them explicitly mentioned in the References.

<sup>2</sup> There are not ‘relativistic effects’ in relativity, as they are not ‘Newtonian effects’ in Newtonian theory. There are defects in the Ptolemaic theory of epicycles that could be corrected by Newtonian theory, and there are defects in Newtonian theory that may be corrected by the theory of relativity. But then their correct appellation is not that of ‘relativistic effects’ but rather that of ‘Newtonian defects’.

systems (some of their coordinate lines, surfaces or hypersurfaces) may be *imagined* as covered by some particles, clocks, rods or radiations submitted to particular motions. But the number of such *physically interpretable* coordinate systems that can be *physically constructed* in practice is strongly limited.

In fact, among the at present physically interpretable coordinate systems, the only one that may be generically constructed<sup>3</sup> is the one based in the Poincaré-Einstein protocol of synchronization, also called *radar system*, which uses two-way light signals from the observer to the events to be coordinated [22]. Unfortunately, this protocol suffers from the bad property of being a retarded protocol (see below). Consequently, in order to increase our knowledge of the physics involved in phenomena depending on the space-time localization of their constituents and, in particular, in making relativistic gravimetry, it is important to learn to construct physically *good* coordinate systems of relativistic quality.

The study of space-time coordinate systems and the diverse protocols associated with their physical construction is a broad and open field in current physics [22]-[28]. Here, we consider those coordinate systems constructed from *relativistic positioning systems*, that are basically defined by four clocks (emitters) broadcasting their respective proper times by means of electromagnetic signals. At each space-time event reached by the signals, the received four times define the *emission coordinates* of this event (with respect to the given positioning system).<sup>4</sup>

## LOCATION SYSTEMS

To clearly differentiate the coordinate system as a mathematical object from its realization as a physical object,<sup>5</sup> it is convenient to characterize this physical object with a proper name. For this reason, the physical object obtained by a peculiar materialization of a coordinate system is called a *location system* [3, 4, 7]. A location system is thus a precise protocol on a particular set of physical fields allowing to materialize a coordinate system.

A location system may have some specific properties [3, 7]. Among them, the more important ones are those of being *generic*, i.e. that can be constructed in any space-time of a given class, (gravity-)*free*, i.e. that the knowledge of the gravitational field is not necessary to construct it,<sup>6</sup> and *immediate*, i.e. that every event may know its coordinates without delay. Thus, for example, location systems based in the Poincaré-Einstein

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<sup>3</sup> Such a statement is not, of course, a *theorem*, because involving real objects, but rather an *epistemic* assertion that results from the analysis of methods, techniques and real and practical possibilities of physical construction of coordinate systems at the present time.

<sup>4</sup> It would be to notice here that a similar construction may be carried out with sub-luminous signals, both in Newtonian and relativistic physics (see [15]). This idea allows us, for example, to develop the Newtonian theory of ultrasonic positioning systems and the corresponding relativistic one.

<sup>5</sup> Different physical protocols, involving different physical fields or different methods to combine them, may be given for a unique mathematical coordinate system.

<sup>6</sup> As a physical object, a location system lives in the physical space-time. In it, even if the metric is not known, such objects as point-like test particles, light rays or signals follow specific paths which, a priori, allow constructing location systems.

synchronization protocol (radar systems) are generic and free, but *not* immediate.

Location systems are usually used either to allow a given observer assigning coordinates to particular events of his environment or to allow every event of a given environment to know its proper coordinates. Location systems constructed for the first of these two functions, following their three-dimensional Newtonian analogues, are called (*relativistic*) *reference systems*. In relativity, where the velocity of transmission of information is finite, they are necessarily not immediate. Poincaré-Einstein location systems are reference systems in the present sense.

Location systems constructed for the second of these two functions which, in addition, are generic, free and immediate, are called [3] (*relativistic*) *positioning systems*. Since Poincaré-Einstein reference systems are the only known location systems but they are not immediate, the first question is if in relativity there exist positioning systems having the three demanded properties of being generic, free and immediate. The epistemic<sup>7</sup> answer is that there exists a little number of them, their paradigmatic representative being constituted by four clocks broadcasting their proper times by means of electromagnetic signals.

In Newtonian physics, when the velocity of transmission of information is supposed infinite, both functions, of reference and positioning, are *exchangeable* in the sense that data obtained from any of the two systems may be transformed in data for the other one. But this is no longer possible in relativity, where the immediate character of positioning systems and the intrinsically retarded character of reference systems imposes a strong decreasing hierarchy. In fact, whereas it is impossible to construct a positioning system starting from a reference system (by transmission of its data), it is always possible and very easy to construct a reference system from a positioning system (it is sufficient that every event sends its coordinate data to the observer).

Consequently, in relativity the experimental or observational context strongly conditions the function, conception and construction of location systems. In addition, by their immediate character, it results that whenever possible, there are positioning systems, and not reference systems, which offer the most interest to be constructed. For the Solar system, it has been recently proposed a ‘galactic’ positioning system, based on the signals of four selected millisecond pulsars and a conventional origin [4]. For the neighborhood of the Earth, a primary, auto-locating, fully relativistic, positioning system has also been proposed, based on four-tuples of satellite clocks broadcasting their proper time as well as the time they receive from the others. The whole constellation of a global navigation satellite system (GNSS), as union of four-tuples of neighboring satellites, constitutes an atlas of local charts for the neighborhood of the Earth, to which a global reference system directly related to the conventional International Celestial Reference System (ICRS) may be associated (SYPO project; see [2, 3, 7, 29]).

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<sup>7</sup> The word *epistemic* is also used here in the sense of the footnote 3.

## RELATIVISTIC POSITIONING SYSTEMS

What is the coordinate system physically realized by four clocks broadcasting their proper time? Every one of the four clocks  $\gamma_A$  (*emitters*) broadcasting his proper time  $\tau^A$ , the future light cones of the points  $\gamma_A(\tau^A)$  constitute the coordinate hypersurfaces  $\tau^A = \text{constant}$  of the coordinate system for some domain of the space-time. At every event of the domain, four of these cones, broadcasting the times  $\tau^A$ , intersect, endowing thus the event with the coordinate values  $\{\tau^A\}$ . In other words, the past light cone of every event cuts the emitter world lines at  $\gamma_A(\tau^A)$ ; then  $\{\tau^A\}$  are the *emission coordinates* of this event.

Let  $\gamma$  be an observer equipped with a receiver allowing the reception of the proper times  $(\tau^A)$  at each point of his trajectory. Then, this observer knows his trajectory in these emission coordinates. We say then that this observer is a *user* of the positioning system. It is worth pointing out that a user could, eventually (but not necessary), carry a clock to measure his proper time  $\tau$ .

A positioning system may be provided with the important quality of being *auto-locating*. For this goal, the emitters have also to be *transmitters* of the proper time they just receive from the other three clocks, so that at every instant they must broadcast their proper time *and* also these other received proper times. Then, any user does not only receive the emitted times  $\{\tau^A\}$  but also the twelve transmitted times  $\{\tau_B^A\}$ . These data allow the user to know the trajectories of the emitter clocks in emission coordinates.

The basic properties of the emission coordinates have been analyzed in [12] for the generic four-dimensional case. As already mentioned, the coordinate system that a positioning system realizes is constituted by four null (one-parametric family of) hypersurfaces, so that its covariant natural frame is constituted by four null 1-forms. Such rather unusual *real null frames* has been considered in the literature but very sparingly.

Zeeman [30] seems the first to have used, for a technical proof, real null frames, and Derrick found them as a particular case of *symmetric frames* [24], later extensively studied by Coll and Morales [26]. They were also those that proved that real null frames constitute a causal class among the 199 possible ones [25]. Coll [23] seems to have been the first to propose the physical construction of coordinate systems by means of light beams, obtaining real null frames as the natural frames of such coordinate systems. Finkelstein and Gibbs [31] proposed symmetric real null frames as a checkerboard lattice for a quantum space-time. The physical construction of relativistic coordinate systems ‘of GPS type’, by means of broadcasted light signals, with a real null coframe as their natural coframe, seems also be first proposed by Coll [1, 2, 3, 7]. Bahder [32] has obtained explicit calculations for the vicinity of the Earth at first order in the Schwarzschild space-time, and Rovelli [27], as representative of a complete set of gauge invariant observables, has developed the case where emitters define a symmetric frame in Minkowski space-time. Blagojević et al. [28] analysed and developed the symmetric frame considered in Finkelstein and Rovelli papers.

References [27, 28, 32], as well as ours [4, 7, 9, 11, 12, 13, 14], have in common the awareness about the need of physically constructible coordinate systems (location systems) in experimental projects concerning relativity. But their future role, as well as their degree of importance with respect to the up to now usual ones, depends on the

authors. For example, Bahder considers them as a way to transmit to any user its coordinates with respect to an exterior, previously given, coordinate system. Nevertheless, our analysis, sketched above, on the generic, free and immediate properties of the relativistic positioning systems lead us to think that they are these systems which are assigned to become the *primary* systems of any precision cartography. Undoubtedly, there is still a lot of work to be made before we be able, as users, to verify and to control this primary character, but the present general state of the theory and the explicit results already known in two, three and four-dimensional space-times encourage this point of view. A key concept for this primary character of a system, although not sufficient, is the already mentioned of auto-location, whose importance in the two-dimensional context has been shown (see next section).

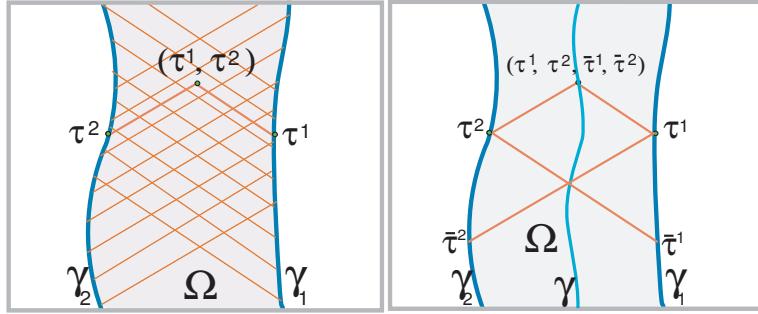
At first glance, relativistic positioning systems are nothing but the relativistic model of the classical GPS (Global Positioning System) but, as explained for example in [7], this is not so. In particular, the GPS uses its emitters (satellites) as simple (and ‘unfortunately moving’!) beacons to transmit *another* spatial coordinate system (the World Geodetic System 84) and an ad hoc time scale (the GPS time), different from the proper time of the embarked clocks, meanwhile for relativistic positioning systems the unsynchronized proper times of the embarked clocks constitute the fundamental ingredients of the system. As sketched in [3] or [7], positioning systems offer a new, paradigmatic, way of decoupling and making independent the spatial segment of the GPS system from its Earth control segment, allowing such a positioning system to be considered as *the primary positioning reference* for the Earth and its environment.

## TWO-DIMENSIONAL APPROACH

A full development of the theory for the generic four-dimensional relativistic positioning systems requires a hard task and a previous training on simple and particular situations. In Coll *et al.* [13] (see also Ferrando [9, 17]) we have presented a two-dimensional approach to relativistic positioning systems introducing the basic features that define them. This two-dimensional approach has the advantage of allowing the use of precise and explicit diagrams which improve the qualitative comprehension of general four-dimensional positioning systems. Moreover, two-dimensional scenarios admit simple and explicit analytic results.

In a two-dimensional space-time we have two emitters  $\gamma_1$  and  $\gamma_2$ . Suppose they broadcast their proper times  $\tau^1$  and  $\tau^2$  by means of electromagnetic signals, and that the signals from each one of the world lines reach the other. The future light cones (here reduced to pairs of ‘light’ lines) cut in the region between both emitters and they are tangent outside. Then, this internal region, bounded by the emitter world lines, defines the *emission coordinate domain*  $\Omega$ . Indeed, the past light cone of every event in  $\Omega$  cuts the emitter world lines at  $\gamma_1(\tau^1)$  and  $\gamma_2(\tau^2)$ , respectively. Then  $(\tau^1, \tau^2)$  are the coordinates of the event (see Fig. 1a).

Emission coordinates are null coordinates and thus, in the two-dimensional case, the space-time metric depends on the sole *metric function*  $m$  and takes the expression:  $ds^2 = m(\tau^1, \tau^2)d\tau^1d\tau^2$ . The plane  $\{\tau^1\} \times \{\tau^2\}$  in which the different data of the positioning system can be transcribed is called the *grid* of the positioning system.



**FIGURE 1.** (a) The region  $\Omega$ , bounded by the emitter world lines, defines the emitter coordinate domain. (b) In an auto-locating positioning system any user  $\gamma$  receives from the emitters the two proper times  $\{\tau^1, \tau^2\}$  and the two transmitted times  $\{\bar{\tau}^1, \bar{\tau}^2\}$ .

Here, a *user* of the positioning system corresponds, according to the above four dimensional definition, to an observer  $\gamma$ , traveling throughout an emission coordinate domain  $\Omega$  and equipped with a receiver allowing the reading of the received proper times  $(\tau^1, \tau^2)$  at each point of his trajectory. Any user receiving continuously the emitted times  $\{\tau^1, \tau^2\}$  knows his trajectory in the grid. Indeed, from these *user positioning data*  $\{\tau^1, \tau^2\}$  the equation  $F$  of the user trajectory may be extracted:  $\tau^2 = F(\tau^1)$ . Analogously, *auto-locating positioning systems* are here systems in which every emitter clock not only broadcasts its proper time but also the proper time that it receives from the other, i.e. the systems that broadcast the emission coordinates of the emitters. Thus, the physical components of an auto-locating positioning system are a *spatial segment* constituted by two emitters  $\gamma_1, \gamma_2$  broadcasting their proper times  $\tau^1, \tau^2$  and the proper times  $\bar{\tau}^2, \bar{\tau}^1$  that they receive each one from the other, and a *user segment* constituted by the set of all users traveling in an internal domain  $\Omega$  and receiving these four broadcast times  $\{\tau^1, \tau^2; \bar{\tau}^1, \bar{\tau}^2\}$  (see Fig. 1b). Then, any user receiving continuously these *emitter positioning data* may also extract from them the equations of the trajectories of the emitters in emission coordinates:  $\varphi_1(\tau^1) = \bar{\tau}^2, \varphi_2(\tau^2) = \bar{\tau}^1$ .

Eventually, the positioning system can be endowed with complementary devices. Thus, the emitters  $\gamma_1, \gamma_2$  could carry accelerometers and broadcast their acceleration  $\alpha_1, \alpha_2$ , meanwhile the users  $\gamma$  could be endowed with receivers able to read, in addition to the emitter positioning data, also the broadcast emitter accelerations  $\{\alpha_1, \alpha_2\}$ . The users can also generate their own data, carrying a clock to measure their proper time  $\tau$  and/or an accelerometer to measure their proper acceleration  $\alpha$ .<sup>8</sup>

The purpose of the (relativistic) theory of positioning systems is to develop the techniques necessary to determine the space-time metric as well as the dynamics of emitters and users by means of physical information carried by the user data  $\{\tau^1, \tau^2; \bar{\tau}^1, \bar{\tau}^2; \alpha_1, \alpha_2; \tau, \alpha\}$ , among others.

In the framework of this two-dimensional approach we have studied a clarifying case: the positioning system defined by two inertial emitters in flat space-time (Coll *et al.* [13]). We have obtained the coordinate change between emitter coordinates and inertial

<sup>8</sup> They can also carry gradiometers, but this situation will be considered elsewhere.

null coordinates and, by using this change, we have shown that the emitter trajectories in the grid are two straight lines with complementary slope. Moreover, we have also found that the emitter positioning data at two events determine the space-time metric in emission coordinates (see Coll *et al.* [13] for more details).

### A. Gravimetry and positioning

The interest of auto-locating positioning systems in gravimetry has been pointed out in Coll *et al.* [13], where we have shown that, if a user has neither a priori information on the gravitational field nor on the positioning system, the user data determine the metric and its first derivatives along the user and emitter trajectories.

In a subsequent work (Coll *et al.* [14]) we have gone further in analyzing the possibility of making relativistic gravimetry or, more generally, the possibility of obtaining the dynamics of the emitters and/or of the user, as well as the detection of the absence or presence of a gravitational field and its measure. This possibility has been examined by means of a (non geodesic) *stationary positioning system*, that is to say a positioning system whose emitters are uniformly accelerated and the radar distance from each one to the other is constant. Such a stationary positioning system is constructed in two different scenarios: Minkowski and Schwarzschild planes.

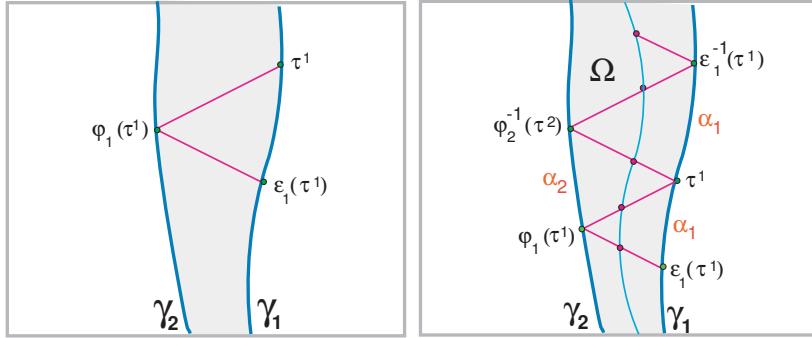
In both scenarios, and for any user, the trajectory of the emitters in the *grid*, i.e. in the plane  $\{\tau^1\} \times \{\tau^2\}$ , are two parallel straight lines. Thus, we find that a stationary positioning system may provide the same emitter positioning data independently of the Schwarzschild mass. At first glance, this fact would seem to indicate the impossibility of extracting dynamical or gravimetric information from them, but this appearance is deceptive.

Indeed, we have proved in Coll *et al.* [14] that the simple qualitative information that the positioning system is stationary (but with no knowledge of the acceleration and mutual radar distance of every emitter) and that the space-time is created by a given mass (but with no knowledge of the particular stationary trajectories followed by the emitters) allows to know the actual accelerations of the emitters, their mutual radar distances and the space-time metric in the region between them in emission coordinates  $\{\tau^i\}$ . The important point for gravimetry is that, in the above context, the data of the Schwarzschild mass may be substituted by that of the acceleration of one of the emitters. Then, besides all the above mentioned results, including the obtaining of the acceleration of the other emitter, the actual Schwarzschild mass of the corresponding space-time may be also calculated.

These relatively simple two-dimensional results strongly suggests that relativistic positioning systems can be useful in gravimetry at least when parameterized models for the gravitational field are used.

### B. Positioning in flat space-time

Above we have considered stationary or geodesic positioning systems where the user has, a priori, a partial or full information about the positioning system. Now we consider a new situation: the user knows the space-time where he is immersed, but he has no information about the positioning system. Can the user data determine the characteristics of the positioning system? Can the user obtain information on his local units of time and



**FIGURE 2.** (a) If an emitter  $\gamma_1$  receives at time  $\tau^1$  a signal after being echoed by the other emitter  $\gamma_2$ , it must be emitted at time  $\epsilon_1(\tau^1)$ ,  $\epsilon_1 = \varphi_2 \circ \varphi_1$ . (b) If a user receives an emitter acceleration in the echo-causal interval  $[\epsilon_1(\tau^1), \tau^1]$ , then he knows all the user data along his trajectory.

distance and his acceleration?

The answer to these questions is still an open problem for a generic space-time, but we have solved it for Minkowski plane. In this flat case we have analyzed the minimum set of data that determines all the user and system information (see Coll *et al.* [33]). From the dynamical equation of the emitters and user we obtain that the user data are not independent quantities: the accelerations of the emitters and of the user along their trajectories are determined by the sole knowledge of the emitter positioning data and of the acceleration of only one of the emitters and only during an *emitter echo*, i.e. the interval between the emission time of a signal by an emitter and its reception time after being reflected by the other emitter (see Fig. 2).

All the results obtained in this two-dimensional approach to relativistic positioning systems show two things. Firstly, the interest of this study in gravimetry and in building a fully relativistic global navigation satellite system (GNSS). Secondly, that a lot of work remains to be done in the development of the general theory, the present one being only one of the first little pieces. In the future, we want to put and solve new two-dimensional problems that improve the qualitative comprehension of the positioning systems, and we want to study the generalization of all these two-dimensional results to the generic four-dimensional case.

## EMISSION COORDINATES IN MINKOWSKI SPACE-TIME

A user of a positioning system that receives the four times  $\{\tau^A\}$  knows its own coordinates in the emission system. Then, if the user wants to know its position in another reference system (for example the ICRS) it is necessary to obtain the relation between both coordinate systems.

Thus, we have the following important problem in relativistic positioning: suppose that the world-lines of the emitters  $\gamma_A(\tau^A)$  are known in a coordinates system  $\{x^\alpha\}$ . Can the user obtain its coordinates in this system if he receives its emission coordinates  $\{\tau^A\}$ ? More precisely, can the coordinate change  $x^\alpha = x^\alpha(\tau^A)$  be obtained?

This coordinate change has been obtained for the case of emitters in particular inertial

motion in flat space-time [34]. Recently, we have solved this problem for a general configuration of the emitters in Minkowski space-time [20, 21]. The transformation  $x^\alpha = x^\alpha(\tau^A)$  between inertial and emitter coordinates is obtained in a covariant way depending on the world-lines  $\gamma_A(\tau^A)$  of the emitters.

In our study, the causal character (time-like, null or space-like) of the emitter configuration plays an important role. The geometric interpretation of this fact is a work in progress. In the short we also want: (i) to study the expression of the coordinate change in a 3+1 formalism adapted to an arbitrary inertial observer, (ii) to particularize our results for emitter motions modeling a satellite constellation around the Earth, and (iii) to analyze the effect of a weak gravitational field on the coordinate transformation.

This current work is necessary to tackle in the future the stated problem for a realistic situation modeling a constellation of a GNSS.

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## REFERENCES

1. B. Coll, "Elements for a theory of relativistic coordinate systems. Formal and physical aspects". In Proceedings of the XXIIIth Spanish Relativity Meeting ERE-2000 on *Reference Frames and Gravitomagnetism*. Eds. J. F. Pascual-Sánchez, L. Floría, A. San Miguel and F. Vicente (World Scientific, Singapore, 2001), 53-65. [<http://coll.cc>].
2. B. Coll, "Physical Relativistic Frames" in *Proceedings Journées Systèmes de Référence Spatio-Temporels, Brussels, 2001*, edited by N. Capitaine (Observatoire de Paris, Paris, France, 2002), 169-174. See also [<http://coll.cc>]. See [15] for the correct causal classification of Newtonian frames in four classes.
3. B. Coll, "A principal positioning system for the Earth" in *Proceedings Journées Systèmes de Référence Spatio-Temporels, Bucharest, 2002*, edited by N. Capitaine and M. Stavinschi (Observatoire de Paris, Paris, France, 2003), 34-38. See also [gr-qc/0306043](http://arxiv.org/abs/gr-qc/0306043).
4. B. Coll and A. Tarantola, "Galactic Positioning System" in *Proceedings Journées Systèmes de Référence Spatio-Temporels, St. Petersburg, 2003*, edited by A. Finkelstein and N. Capitaine (Institute of Applied Astronomy of Russian Academy of Science, St. Petersburg, Russia, 2004), 333-334. A poster is available in <http://coll.cc>. See also [arXiv:0905.4121\[gr-qc\]](http://arxiv.org/abs/0905.4121).
5. B. Coll and J. J. Martín, (eds.) Proc. Int. School on *Relativistic Coordinates, Reference and Positioning Systems*. Salamanca, Spain, Jaunary 2005. Universidad de Salamanca (in press).
6. A. Tarantola, L. Klimes, J. M. Pozo and B. Coll, "Gravimetry, Relativity, and the Global Navigation Satellite Systemst' in Proc. Int. School on *Relativistic Coordinates, Reference and Positioning Systems*, B. Coll and J. J. Martín, (eds.) Salamanca, Spain, Jaunary 2005. Universidad de Salamanca (in press). See also [arXiv:0905.3798\[gr-qc\]](http://arxiv.org/abs/0905.3798).
7. B. Coll, "Relativistic Positioning Systems" in *Proceeding of the XXVIII Spanish Relativity Meeting ERE-2005 on A Century of Relativity Physics*, AIP Conf. Proc. Vol. 841 (AIP, New York, 2006), 277-284. See also [gr-qc/0601110](http://arxiv.org/abs/gr-qc/0601110).
8. J. M. Pozo, "Emission coordinates and the Central Observer" in *Proceeding of the XXVIII Spanish Relativity Meeting ERE-2005 on A Century of Relativity Physics*, AIP Conf. Proc. Vol. 841 (AIP, New York, 2006), 641-642.
9. J. J. Ferrando, "Coll Positioning systems: a two-dimensional approach" in *Proceeding of the XXVIII Spanish Relativity Meeting ERE-2005 on A Century of Relativity Physics*, AIP Conf. Proc. Vol. 841 (AIP, New York, 2006), 424-427. See also [gr-qc/0601117](http://arxiv.org/abs/gr-qc/0601117).
10. J. A. Morales Lladosa, "Coordinates and frames from the causal point of view" in *Proceeding of the XXVIII Spanish Relativity Meeting ERE-2005 on A Century of Relativity Physics*, AIP Conf. Proc. Vol. 841 (AIP, New York, 2006), 537-541. See also [gr-qc/0601138](http://arxiv.org/abs/gr-qc/0601138).

11. J.M. Pozo, "Some properties of emission coordinates" in *Proceedings Journées Systèmes de Référence Spatio-Temporels, Warsaw, 2005*, edited by A. Brzeziński, N. Capitaine and B. Kulaczek (Space Research Centre PAS, Warsaw, Poland, 2006), 286-289. See also gr-qc/0601125.
12. B. Coll and J. M. Pozo, *Class. Quantum. Grav.* **23**, 7395 (2006).
13. B. Coll, J. J. Ferrando and J. A. Morales, *Phys. Rev. D* **73**, 084017 (2006).
14. B. Coll, J. J. Ferrando and J. A. Morales, *Phys. Rev. D* **74**, 104003 (2006).
15. B. Coll, J. J. Ferrando and J. A. Morales, "On Newtonian frames". See arXiv:0707.0248[gr-qc].
16. J. A. Morales Lladosa, "Newtonian and Relativistic Location Systems" in *Proceeding of the XXX Spanish Relativity Meeting ERE-2007 on Relativistic Astrophysics and Cosmology*, A. Oscoz, E. Mediavilla and M. Serra-Ricard (eds.) EAS Publications Series, **30**, 119-124, (2008).
17. J. J. Ferrando, "Positioning in flat two-dimensional space-time" in *Proceeding of the XXX Spanish Relativity Meeting ERE-2007 on Relativistic Astrophysics and Cosmology*, A. Oscoz, E. Mediavilla and M. Serra-Ricard (eds.) EAS Publications Series, **30**, 323-327, (2008).
18. B. Coll, J. J. Ferrando and J. A. Morales-Lladosa, "Four causal classes of Newtonian frames" (submitted to *Foundations of Physics*, 3 Mars 2009).
19. B. Coll, J. J. Ferrando and J. A. Morales-Lladosa, "Newtonian and relativistic emission coordinates" (submitted to *Phys. Rev. D*, 10 May 2009).
20. B. Coll, J. J. Ferrando and J. A. Morales, "Emission coordinates in Minkowski space-time" in *Proceedings of the XXXIth Spanish Relativity Meeting ERE-2008 on Physics and Mathematics of Gravitation*, K. E. Kunze, M. Mars and M. A. Vázquez-Mozo (eds.) AIP Conference Proceedings, Melville, N.Y. No. 1122 (AIP, New York, 2009), 225-228.
21. B. Coll, J. J. Ferrando and J. A. Morales-Lladosa, "Positioning systems in Minkowski space-time: emission coordinates" (in preparation).
22. V. Perlick, "On the Radar Method in General-Relativistic Spacetimes". In *Lasers, Clocks, and Drag-Free Control. Exploration of Relativistic Gravity in Space*. Eds. H. Dittus, C. Lämmerzahl, S. G. Turyshev (Springer, Berlin, 2007), 131-152. See also arXiv:0708.0170[gr-qc].
23. B. Coll, "Coordenadas Luz en Relatividad" in *Trobades Científiques de la Mediterrània: Actes E.R.E-85*, edited by A. Molina (Servei de Publicacions de l'ETSEIB, Barcelona, 1985), 29-38. An English translation of this article entitled *Light coordinates in relativity* is available in <http://www.coll.cc/>.
24. G. H. Derrick, *J. Math. Phys.* **22**, 2896 (1981).
25. B. Coll, and J. A. Morales, *Int. Jour. Theor. Phys.* **31**, 1045 (1992).
26. B. Coll and J. A. Morales, *J. Math. Phys.* **32**, 2450 (1992).
27. C. Rovelli, *Phys. Rev. D* **65**, 044017 (2002). See also gr-qc/0110003.
28. M. Blagojević, J. Garecki, F. W. Hehl and Y. N. Obukhov, *Phys. Rev. D* **65**, 044018 (2002). See also gr-qc/0110078.
29. J.-F. Pascual-Sánchez, *Ann. Phys. (Leipzig)* **16**, 258 (2007).
30. E.C. Zeeman, *Jour. of Math. Phys.* **5**, 490 (1964).
31. D. Finkelstein, and J. Michael Gibbs, *Intern. Jour. of Theor. Phys.* **32**, 1801 (1993); D. Finkelstein, *Quantum Relativity*, (Springer-Verlag, Berlin, 1994).
32. T.B. Bahder, *Am. J. Phys.* **69**, 315 (2001).
33. B. Coll, J. J. Ferrando and J. A. Morales-Lladosa, "Positioning in flat two-dimensional space-time" (in preparation).
34. D. Bini, A. Geralico, M. L. Ruggiero and A. Tartaglia, *Class. Quantum. Grav.* **25**, 205011 (2008).